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Successive Superalgebraic Truncations from the Four-Dimensional Maximal Supergravity

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ABSTRACT

We study the four-dimensional $N=8$ maximal supergravity in the context of Lie superalgebra $SU(8/1)$. All possible successive superalgebraic truncations from four-dimensional $N=8$ theory to $N=7, 6, \dots, 1$ supergravity theories are systematically realized as sub-superalgebra chains of $SU(8/1)$ by using the Kac-Dynkin weight techniques.

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I. Introduction

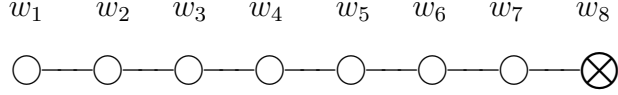
There have been considerable interests in superalgebras[1,2] which are relevant to superunifications[3], nuclear physics[4], supergravities[5], and superstring theories[6]. Supersymmetric extensions of Poincaré algebra in D -dimensional space-time were reviewed, and their representations (reps) for the supermultiplets of all known supergravity theories were extensively searched by Strathdee[7]. This work has been an extremely useful guideline for studying supergravity. Reps of supermultiplets have been obtained within the framework of the little algebras of the super-Poincaré algebra. Fermionic generators, which produce supertranslations, are adjoined to the algebra of the Poincaré group. Cremmer[8] developed the method for consistent truncations by choosing a particular rep of real symplectic metric in order to derive $N=6,4,2$ supergravities from $N=8$ in five dimensions. But this method is too complicated.

On the other hand, during several years, we have shown that superalgebras allow a more systematic analysis for finding the supermultiplets [9,10] of several supergravity and type-IIB closed superstring theories by using the Kac-Dynkin weight techniques of the $SU(m/n)$ Lie superalgebra[11]. Recently, we have shown that the massless reps of supermultiplets of the $D=10$, $N=2$ chiral supergravity[12] and the $D=4$, $N=8$ supergravity[13] belong to only one irreducible representation (irrep) of the $SU(8/1)$ superalgebra using the Kac-Dynkin method[14].

In this paper, we show that the successive superalgebraic truncations from the $D=4$, $N=8$ maximal supergravity[13] to $N=7, 6, \dots, 1$ theories can be systematically realized as sub-superalgebra chains of $SU(8/1)$ Lie superalgebra by using projection matrices[15]. In Sec. II, we briefly recapitulate the mathematical structure of the $SU(8/1)$ superalgebra related to the $D=4$, $N=8$ maximal supergravity. In Sec. III, we explicitly show that all possible supermultiplets of $D=4$, $N=7, 6, \dots, 1$ theories can be systematically obtained from this maximal supergravity by successive superalgebraic truncations. The last section contains conclusions.

II. SU(8/1) Superalgebra with the Kac-Dynkin Method

The Kac-Dynkin diagram of the SU(8/1) Lie superalgebra is



where the set $(w_1 \ w_2 \ \cdots \ w_8)$ characterizes the highest weight vector of an irrep[1,2]. A weight component w_i ($i \neq 8$) should be a nonnative integer, while w_8 can be any complex number. The seven white nodes in the Kac-Dynkin diagram denote the simple even roots α_i ($i = 1, 2, \dots, 7$), which constitute SU(8) subalgebra, while the last node denotes the simple odd root β_8 .

The corresponding graded Cartan matrix is

$$\begin{bmatrix} 2 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 2 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 2 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \end{bmatrix}. \quad (1)$$

Each simple even root α_i corresponds to the i -th column of the graded Cartan matrix, while the simple odd root β_8 corresponds to the last column of the graded Cartan matrix. Then, the positive and negative simple roots α_i^\pm and β_8^\pm are directly read from the graded Cartan matrix as follows;

$$\begin{aligned} \alpha_1^\pm &= (\pm 2 \ \mp 1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0), \\ \alpha_2^\pm &= (\mp 1 \ \pm 2 \ \mp 1 \ 0 \ 0 \ 0 \ 0 \ 0), \\ &\vdots \\ \alpha_7^\pm &= (0 \ 0 \ 0 \ 0 \ 0 \ \mp 1 \ \pm 2 \ \mp 1), \\ \beta_8^\pm &= (0 \ 0 \ 0 \ 0 \ 0 \ 0 \ \mp 1 \ 0). \end{aligned} \quad (2)$$

On the other hand, other odd roots are easily obtained by

$$\beta_i^\pm = [\alpha_i^\pm, \beta_{i+1}^\pm], \quad i = 1, 2, \dots, 7. \quad (3)$$

Note that the action by an odd root β_i^\pm alternates a bosonic floor with a fermionic one.

The fundamental rep of $SU(8/1)$ is $(1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0)$, which is $[(\mathbf{8}, \mathbf{1})_F \oplus (\mathbf{1}, \mathbf{1})_B]$ in the $SU(8) \otimes U(1)$ bosonic subalgebra basis where the subscripts F and B stand for fermionic and bosonic degrees of freedom, respectively, as follows:

$$\begin{aligned} & (1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0) \\ | \text{gnd} > & (1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0) = (\mathbf{8}, \mathbf{1}) \\ & \downarrow \beta_1^- \\ | \text{1st} > & (0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 1) = (\mathbf{1}, \mathbf{1}). \end{aligned} \quad (4)$$

In contrast to the usual Lie algebra $SU(n)$, the complex conjugate rep of the fundamental rep is given by $(0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ -1) = [(\mathbf{1}, \mathbf{1})_B \oplus (\overline{\mathbf{8}}, \mathbf{1})_F]$:

$$\begin{aligned} & (0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ -1) \\ | \text{gnd} > & (0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ -1) = (\mathbf{1}, \mathbf{1}) \\ & \downarrow \beta_8^- \\ | \text{1st} > & (0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 1 \ -1) = (\overline{\mathbf{8}}, \mathbf{1}). \end{aligned} \quad (5)$$

Note that the last component of the highest weight vector of $SU(8/1)$ relating with the simple odd root can be any *complex* number. Similar to the usual Lie algebras, we have the following relation from the tensor product of the above two reps in Eqs.(4) and (5):

$$(1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0) \otimes (0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ -1) = (1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ -1) \oplus (0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0). \quad (6)$$

Then, one can easily recognize that the even and odd roots generate the adjoint rep $(1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ -1)$ of $SU(8/1)$ as follows:

$$\begin{aligned}
& (1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ -1) \\
| \text{gnd} > & (1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ -1) = \beta_i^+ \\
| \text{1st} > & \begin{aligned} (1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 1 \ -1) &= \text{SU}(8) \\ (0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0) &= \text{U}(1) \end{aligned} \\
| \text{2nd} > & (0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 1 \ 0) = \beta_i^-.
\end{aligned} \tag{7}$$

Now let us consider the irreps of $\text{SU}(8/1)$ superalgebra. There are two types of irreps, which are *typical* and *atypical* [1,11,16]. All atypical reps of $\text{SU}(8/1)$ are characterized by the last component of the highest weight which corresponds to the last node of the Kac-Dynkin diagram. The atypicality condition[11] is given by

$$w_8 = - \sum_{j=i}^7 w_j + i - 8, \quad 1 \leq i \leq 8. \tag{8}$$

Since an odd root β_i^\pm string is terminated in the full weight system for the case of atypical rep, which w_8 satisfies Eq.(8) for a specific i , the atypical reps generally have not equal bosonic and fermionic degrees of freedom.

On the other hand, typical reps of $\text{SU}(8/1)$ consist of nine floors due to the existances of the eight odd roots, and have equal bosonic and fermionic degrees of freedom. The lowest dimensional typical rep is $(0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ w_8) = [\mathbf{128}_B \oplus \mathbf{128}_F]$ for $w_8 \neq 0, -1, -2, -3, -4, -5, -6$, and -7 such as

$$\begin{aligned}
& (0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ w_8) \\
| \text{gnd} > & (0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ w_8) = \mathbf{1} \\
& \quad \downarrow \beta_8^- \\
| \text{1st} > & (0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 1 \ w_8) = \overline{\mathbf{8}} \\
& \quad \downarrow \beta_7^- \\
| \text{2nd} > & (0 \ 0 \ 0 \ 0 \ 0 \ 1 \ 0 \ w_8 + 1) = \overline{\mathbf{28}} \\
& \quad \downarrow \beta_6^- \\
| \text{3rd} > & (0 \ 0 \ 0 \ 0 \ 1 \ 0 \ 0 \ w_8 + 2) = \overline{\mathbf{56}} \\
& \quad \downarrow \beta_5^- \\
| \text{4th} > & (0 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0 \ w_8 + 3) = \mathbf{70} \\
& \quad \downarrow \beta_4^- \\
| \text{5th} > & (0 \ 0 \ 1 \ 0 \ 0 \ 0 \ 0 \ w_8 + 4) = \mathbf{56} \\
& \quad \downarrow \beta_3^- \\
| \text{6th} > & (0 \ 1 \ 0 \ 0 \ 0 \ 0 \ 0 \ w_8 + 5) = \mathbf{28} \\
& \quad \downarrow \beta_2^- \\
| \text{7th} > & (1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ w_8 + 6) = \mathbf{8} \\
& \quad \downarrow \beta_1^- \\
| \text{8th} > & (0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ w_8 + 7) = \mathbf{1}.
\end{aligned} \tag{9}$$

Particularly taking $w_8 = -\frac{7}{2}$, the weight system in Eq.(9) shows the *typical* and *real* property. By using these properties, we have already shown that the typical rep $(0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ -\frac{7}{2})$ is beautifully identified with the supermultiplets of the $D=4$, $N=8$ supergravity and $D=10$, $N=2$ chiral supergravity[12]. Note that the rep with $w_8 = 0$ in Eq.(9) is reduced to a singlet rep $(0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0)$ because the odd root β_8^- string is terminated due to the atypicality condition (8). One can also generate the atypical rep $(0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ -1)$, which is the complex conjugate rep of the fundamental rep, in Eq.(5) from Eq.(9) since the β_7^- string is broken for $w_8 = -1$.

The next higher dimensional typical rep is $(1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ w_8) = [\mathbf{1024}_B \oplus \mathbf{1024}_F]$ for $w_8 \neq 0, -1, -2, -3, -4, -5, -6$, and -8 . Note that the fundamental rep with $w_8 = 0$ in Eq.(4) and the adjoint rep with $w_8 = -1$ in Eq.(7) are atypical because the β_2^- and β_8^- strings are terminated from the typical rep $(1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ w_8)$, respectively.

III. Successive Superalgebraic Truncations from $N=8$ to $N=7,6,\dots,1$ in $D=4$

The massless reps of supermultiplets for $D=4$, $N=8$ supergravity[7, 13] have the algebraic structure of $SU(8)\otimes SO(2)$, where $SU(8)$ is the supersymmetry algebra and $D=4$ light-cone symmetry is $SO(2)\approx U(1)$, *i.e.*, the helicity quantum number is given by the eigenvalue of this $U(1)$ generator. Since the bosonic subalgebra of $SU(8/1)$ is $SU(8)\otimes U(1)$, we have recently shown that the well-known $D=4$, $N=8$ supermultiplets correspond to only one *real* typical irrep with $w_8 = -\frac{7}{2}$ in Eq.(9), which is

$$(0\ 0\ 0\ 0\ 0\ 0\ 0\ -\frac{7}{2}) = [\mathbf{1}_B \oplus \overline{\mathbf{8}}_F \oplus \overline{\mathbf{28}}_B \oplus \overline{\mathbf{56}}_F \oplus \mathbf{70}_B \oplus \mathbf{56}_F \oplus \mathbf{28}_B \oplus \mathbf{8}_F \oplus \mathbf{1}_B]. \quad (10)$$

Note that the helicity state of each multiplet crucially depends upon the condition $w_8 = -\frac{7}{2}$, which guarantees equal degrees of freedom for bosons and fermions. The regular branching $SU(n/1)\rightarrow SU(n)\otimes U(1)$ is attained by removing the last node from the Kac-Dynkin diagram of $SU(n/1)$, and the corresponding projection matrix $\mathbf{P}_1(n)$ is given by

$$\mathbf{P}_1(n) = \left[\begin{array}{cccc|c} 1 & 0 & \cdots & 0 & 1 \\ 0 & 1 & 0 & \vdots & 2 \\ \vdots & 0 & \ddots & 0 & \vdots \\ \vdots & \vdots & 0 & 1 & n-1 \\ 0 & \cdots & \cdots & 0 & n \end{array} \right]. \quad (11)$$

By letting $\mathbf{P}_1(8)$ act on the whole weight system of $SU(8/1)$, and normalizing the $U(1)$ eigenvalues by -14 , one gets the desired supermultiplets as follows:

floor	SU(8/1)	field	helicity
gnd >	(0 0 0 0 0 0 0 $-\frac{7}{2}$)	e_μ^a	+2
1st >	(0 0 0 0 0 0 1 $-\frac{7}{2}$)	$\bar{8}\Psi_\mu$	$+\frac{3}{2}$
2nd >	(0 0 0 0 0 1 0 $-\frac{5}{2}$)	$\bar{28}A_\mu$	+1
3rd >	(0 0 0 0 1 0 0 $-\frac{3}{2}$)	$\bar{56}\lambda$	$+\frac{1}{2}$
4th >	(0 0 0 1 0 0 0 $-\frac{1}{2}$)	70ϕ	0
5th >	(0 0 1 0 0 0 0 $+\frac{1}{2}$)	56λ	$-\frac{1}{2}$
6th >	(0 1 0 0 0 0 0 $+\frac{3}{2}$)	$28A_\mu$	-1
7th >	(1 0 0 0 0 0 0 $+\frac{5}{2}$)	$8\Psi_\mu$	$-\frac{3}{2}$
8th >	(0 0 0 0 0 0 0 $+\frac{7}{2}$)	e_μ^a	-2.

(12)

Now let us consider successive superalgebraic truncations from the $D=4$, $N=8$ maximal theory to the $N=7, 6, \dots, 1$ supergravities. One must carefully remove extra gravitino multiplets in a consistent manner to guarantee the existence of the $N=7, 6, \dots, 1$ theories. We can easily show that superalgebraic truncations is systematically understood as a regular branching chain of SU(8/1):

$$\overbrace{\text{SU}(8/1)}^{N=8} \rightarrow \overbrace{\text{SU}(7/1) \otimes \text{U}(1)}^{N=7} \rightarrow \overbrace{\text{SU}(6/1) \otimes [\text{U}(1)]^2}^{N=6} \rightarrow \dots \rightarrow \overbrace{[\text{U}(1)]^8}^{N=1} \quad (13)$$

This branching chain is systematically attained by successively removing the first node from the corresponding Kac-Dynkin diagrams. The U(1) supercharge assignment for the branching pattern $\text{SU}(N/1) \rightarrow \text{SU}(N-1/1) \otimes \text{U}(1)$ for $N \geq 2$ is given by

$$\text{U}(1) = \text{diag.}(N-2, \overbrace{-1, -1, \dots, -1}^{N-\text{terms}}) \quad (14)$$

since Eq.(14) must satisfy the supertraceless condition[11]. The corresponding projection matrix $\mathbf{P}_2(n)$ is given by

$$\mathbf{P}_2(n) = \left[\begin{array}{cccc|c} 0 & \cdots & \cdots & \cdots & 0 & | & n-2 \\ 1 & 0 & \vdots & \vdots & \vdots & | & n-3 \\ 0 & 1 & 0 & \vdots & \vdots & | & \vdots \\ \vdots & 0 & \ddots & 0 & \vdots & | & 1 \\ \vdots & \vdots & 0 & 1 & 0 & | & 0 \\ 0 & \cdots & \cdots & 0 & 1 & | & -1 \end{array} \right]. \quad (15)$$

Then we are ready to analyse successive superalgebraic truncations.

3.1 $N = 7$ Supergravity

First, let us consider the consistent superalgebraic truncation from the $D=4$, $N=8$ to the $D=4$, $N=7$ theory. By allowing the matrix $\mathbf{P}_2(8)$ to act on the right hand side of the $SU(8/1)$ weight system in Eq.(12) that describes of the supermultiplets of the $D=4$, $N=8$ theory, one easily gets the two desired reducible $SU(7/1) \otimes U^a(1)$ reps for $N=7$ as follows:

$$(0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ -\frac{7}{2}) \rightarrow (0 \ 0 \ 0 \ 0 \ 0 \ 0 \ -\frac{7}{2})(\frac{7}{2}) \oplus (0 \ 0 \ 0 \ 0 \ 0 \ 0 \ -\frac{5}{2})(-\frac{7}{2}). \quad (16)$$

Since the complex conjugate rep of $(0 \ 0 \ 0 \ 0 \ 0 \ 0 \ -\frac{7}{2})(\frac{7}{2})$ is $(0 \ 0 \ 0 \ 0 \ 0 \ 0 \ -\frac{5}{2})(-\frac{7}{2})$, the two combined reps in Eq.(16) still maintain the desired real property although the truncation is occurred.

Then one can easily recognize that the typical rep $(0 \ 0 \ 0 \ 0 \ 0 \ 0 \ -\frac{7}{2})$ of $SU(7/1)$ with the $U^a(1)$ quantum number $h^a = \frac{7}{2}$ is $[\mathbf{64}_B \oplus \mathbf{64}_F]_{\uparrow}$, which contains the half of the supermultiplets of the $N=7$ theory from the branching rule $SU(7/1) \rightarrow SU(7) \otimes U^b(1)$ by using the projection matrix $\mathbf{P}_1(7)$ as follows:

floor	SU(7/1)	field	helicity
gnd >	(0 0 0 0 0 0 $-\frac{7}{2}$)	e_μ^a	+2
1st >	(0 0 0 0 0 1 $-\frac{7}{2}$)	$\bar{7}\Psi_\mu$	$+\frac{3}{2}$
2nd >	(0 0 0 0 1 0 $-\frac{5}{2}$)	$\bar{21}A_\mu$	+1
3rd >	(0 0 0 1 0 0 $-\frac{3}{2}$)	$\bar{35}\lambda$	$+\frac{1}{2}$
4th >	(0 0 1 0 0 0 $-\frac{1}{2}$)	35ϕ	0
5th >	(0 1 0 0 0 0 $+\frac{1}{2}$)	21λ	$-\frac{1}{2}$
6th >	(1 0 0 0 0 0 $+\frac{3}{2}$)	$7A_\mu$	-1
7th >	(0 0 0 0 0 0 $+\frac{5}{2}$)	Ψ_μ	$-\frac{3}{2}$,

(17)

where the desired helicity quantum number h is given by

$$h = -\frac{1}{84}h^a - \frac{1}{12}h^b. \quad (18)$$

The remaining half supermultiplets are identified with the rep $(0 0 0 0 0 0 -\frac{5}{2}) = [\mathbf{64}_B \oplus \mathbf{64}_F]_\downarrow$ with $h^a = -\frac{7}{2}$ as follows:

floor	SU(7/1)	field	helicity
gnd >	(0 0 0 0 0 0 $-\frac{5}{2}$)	Ψ_μ	$+\frac{3}{2}$
1st >	(0 0 0 0 0 1 $-\frac{5}{2}$)	$\bar{7}A_\mu$	+1
2nd >	(0 0 0 0 1 0 $-\frac{3}{2}$)	$\bar{21}\lambda$	$+\frac{1}{2}$
3rd >	(0 0 0 1 0 0 $-\frac{1}{2}$)	$\bar{35}\phi$	0
4th >	(0 0 1 0 0 0 $+\frac{1}{2}$)	35λ	$-\frac{1}{2}$
5th >	(0 1 0 0 0 0 $+\frac{3}{2}$)	$21A_\mu$	-1
6th >	(1 0 0 0 0 0 $+\frac{5}{2}$)	$7\Psi_\mu$	$-\frac{3}{2}$
7th >	(0 0 0 0 0 0 $+\frac{7}{2}$)	e_μ^a	-2.

(19)

Note that the upper (lower) arrow indicates that the multiplet contains the graviton mode having the positive (negative) helicity. Therefore, the well-known supergravity multiplets $[\mathbf{128}_B \oplus \mathbf{128}_F]$ of the $D=4$, $N=7$ theory[7] given by

$$\begin{aligned} & [\mathbf{1}_2 \oplus \overline{\mathbf{7}}_{3/2} \oplus \overline{\mathbf{21}}_1 \oplus \overline{\mathbf{35}}_{1/2} \oplus \mathbf{35}_0 \oplus \mathbf{21}_{-1/2} \oplus \mathbf{7}_{-1} \oplus \mathbf{1}_{-3/2}] \\ & \oplus [\mathbf{1}_{3/2} \oplus \overline{\mathbf{7}}_1 \oplus \overline{\mathbf{21}}_{1/2} \oplus \overline{\mathbf{35}}_0 \oplus \mathbf{35}_{-1/2} \oplus \mathbf{21}_{-1} \oplus \mathbf{7}_{-3/2} \oplus \mathbf{1}_{-2}], \end{aligned} \quad (20)$$

where the states are represented by their $SU(7)$ dimensionality and $U(1)$ helicity, are exactly identified with two combined irreps of $SU(7/1)$ in Eqs.(17) and (19). Note that each ground floor in Eqs.(17) and (19) originates from the ground floor and the first floor of the $SU(8/1)$ rep in Eq.(12), respectively.

3.2 $N=6$ Supergravity

By using the projection matrix $\mathbf{P}_2(7)$ to act on the right hand side of the $SU(7/1)$ weight system in Eq.(16), one gets the following four reducible $SU(6/1) \otimes U^c(1)$ reps which are appropriate to $N=6$ theory:

$$(0 \ 0 \ 0 \ 0 \ 0 \ 0 \ -\frac{7}{2}) \rightarrow (0 \ 0 \ 0 \ 0 \ 0 \ -\frac{7}{2})(\frac{7}{2}) \oplus (0 \ 0 \ 0 \ 0 \ 0 \ -\frac{5}{2})(-\frac{5}{2}), \quad (21)$$

$$(0 \ 0 \ 0 \ 0 \ 0 \ 0 \ -\frac{5}{2}) \rightarrow (0 \ 0 \ 0 \ 0 \ 0 \ -\frac{5}{2})(\frac{5}{2}) \oplus (0 \ 0 \ 0 \ 0 \ 0 \ -\frac{3}{2})(-\frac{7}{2}). \quad (22)$$

Then, we can easily show that the typical rep $(0 \ 0 \ 0 \ 0 \ 0 \ -\frac{7}{2}) = [\mathbf{32}_B \oplus \mathbf{32}_F]_{\uparrow}$ with $h^a = h^c = \frac{7}{2}$ contains the half of the supermultiplets of the $N=6$ theory from the branching rule $SU(6/1) \rightarrow SU(6) \otimes U^d(1)$ by using $\mathbf{P}_1(6)$ as follows:

floor	SU(6/1)	field	helicity
$ \text{gnd} >$	$(0 \ 0 \ 0 \ 0 \ 0 \ -\frac{7}{2})$	e_μ^a	$+2$
$ \text{1st} >$	$(0 \ 0 \ 0 \ 0 \ 1 \ -\frac{7}{2})$	$\bar{6}\Psi_\mu$	$+\frac{3}{2}$
$ \text{2nd} >$	$(0 \ 0 \ 0 \ 1 \ 0 \ -\frac{5}{2})$	$\bar{15}A_\mu$	$+1$
$ \text{3rd} >$	$(0 \ 0 \ 1 \ 0 \ 0 \ -\frac{3}{2})$	20λ	$+\frac{1}{2}$
$ \text{4th} >$	$(0 \ 1 \ 0 \ 0 \ 0 \ -\frac{1}{2})$	15ϕ	0
$ \text{5th} >$	$(1 \ 0 \ 0 \ 0 \ 0 \ +\frac{1}{2})$	6λ	$-\frac{1}{2}$
$ \text{6th} >$	$(0 \ 0 \ 0 \ 0 \ 0 \ +\frac{3}{2})$	A_μ	$-1,$

(23)

where the desired helicity quantum number h is

$$h = -\frac{1}{84}h^a - \frac{1}{60}h^c - \frac{1}{10}h^d. \quad (24)$$

And the other half of the supermultiplets are in the complex conjugate rep $(0 \ 0 \ 0 \ 0 \ 0 \ -\frac{3}{2}) = [\mathbf{32}_B \oplus \mathbf{32}_F]_\downarrow$ in Eq.(21) with the quantum numbers $h^a = h^c = -\frac{7}{2}$, which is the graviton multiplet with the helicity -2 as follows:

floor	SU(6/1)	field	helicity
$ \text{gnd} >$	$(0 \ 0 \ 0 \ 0 \ 0 \ -\frac{3}{2})$	A_μ	$+1$
$ \text{1st} >$	$(0 \ 0 \ 0 \ 0 \ 1 \ -\frac{3}{2})$	$\bar{6}\lambda$	$+\frac{1}{2}$
$ \text{2nd} >$	$(0 \ 0 \ 0 \ 1 \ 0 \ -\frac{1}{2})$	$\bar{15}\phi$	0
$ \text{3rd} >$	$(0 \ 0 \ 1 \ 0 \ 0 \ +\frac{1}{2})$	20λ	$-\frac{1}{2}$
$ \text{4th} >$	$(0 \ 1 \ 0 \ 0 \ 0 \ +\frac{3}{2})$	$15A_\mu$	-1
$ \text{5th} >$	$(1 \ 0 \ 0 \ 0 \ 0 \ +\frac{5}{2})$	$6\Psi_\mu$	$-\frac{3}{2}$
$ \text{6th} >$	$(0 \ 0 \ 0 \ 0 \ 0 \ +\frac{7}{2})$	e_μ^a	$-2.$

(25)

Note that the rest two typical reps $(0 \ 0 \ 0 \ 0 \ 0 \ -\frac{5}{2})$ carrying $U^c(1) = \pm\frac{5}{2}$ are extra gravitino multiplets, which should be removed for consistency in the $N = 6$ theory.

As a result, the desired supergravity multiplets $[\mathbf{64}_B \oplus \mathbf{64}_F]$ of the $D=4$, $N=6$ theory, which is given by

$$\begin{aligned} & [\mathbf{1}_2 \oplus \overline{\mathbf{6}}_{3/2} \oplus \overline{\mathbf{15}}_1 \oplus \mathbf{20}_{1/2} \oplus \mathbf{15}_0 \oplus \mathbf{6}_{-1/2} \oplus \mathbf{1}_{-1}] \\ & \oplus [\mathbf{1}_1 \oplus \overline{\mathbf{6}}_{1/2} \oplus \overline{\mathbf{15}}_0 \oplus \mathbf{20}_{-1/2} \oplus \mathbf{15}_{-1} \oplus \mathbf{6}_{-3/2} \oplus \mathbf{1}_{-2}], \end{aligned} \quad (26)$$

are exactly identified with two irreps of $SU(6/1)$ in Eqs.(23) and (25).

3.3 $N = 5$ Supergravity

Next, let us carry out superalgebraic truncations from the $N=6$ to the $N=5$ theory. By using the projection matrix $\mathbf{P}_2(6)$, one easily gets $SU(5/1) \otimes U^e(1)$ reducible reps for $N=5$ as follows:

$$(0 \ 0 \ 0 \ 0 \ 0 \ -\frac{7}{2}) \rightarrow (0 \ 0 \ 0 \ 0 \ -\frac{7}{2})(\frac{7}{2}) \oplus (0 \ 0 \ 0 \ 0 \ -\frac{5}{2})(-\frac{3}{2}) \quad (27)$$

$$(0 \ 0 \ 0 \ 0 \ 0 \ -\frac{3}{2}) \rightarrow (0 \ 0 \ 0 \ 0 \ -\frac{3}{2})(\frac{3}{2}) \oplus (0 \ 0 \ 0 \ 0 \ -\frac{1}{2})(-\frac{7}{2}) \quad (28)$$

The typical rep $(0 \ 0 \ 0 \ 0 \ -\frac{7}{2}) = [\mathbf{16}_B \oplus \mathbf{16}_F]_{\uparrow}$ with $h^a = h^c = h^e = \frac{7}{2}$ is relevant to the graviton multiplet for the $N=5$ theory when $SU(5/1) \rightarrow SU(4/1) \otimes U^f(1)$ as follows

floor	SU(5/1)	field	helicity	
$ \text{gnd} >$	$(0 \ 0 \ 0 \ 0 \ -\frac{7}{2})$	e_{μ}^a	$+2$	
$ \text{1st} >$	$(0 \ 0 \ 0 \ 1 \ -\frac{7}{2})$	$\overline{\mathbf{5}}\Psi_{\mu}$	$+\frac{3}{2}$	
$ \text{2nd} >$	$(0 \ 0 \ 1 \ 0 \ -\frac{5}{2})$	$\overline{\mathbf{10}}A_{\mu}$	$+1$	(29)
$ \text{3rd} >$	$(0 \ 1 \ 0 \ 0 \ -\frac{3}{2})$	10λ	$+\frac{1}{2}$	
$ \text{4th} >$	$(1 \ 0 \ 0 \ 0 \ -\frac{1}{2})$	5ϕ	0	
$ \text{5th} >$	$(0 \ 0 \ 0 \ 0 \ +\frac{1}{2})$	λ	$-\frac{1}{2}$,	

where the helicity quantum number h is

$$h = -\frac{1}{84}h^a - \frac{1}{60}h^c - \frac{1}{40}h^e - \frac{1}{8}h^f. \quad (30)$$

The remaining half of the supermultiplets are identified with $(0 \ 0 \ 0 \ 0 \ -\frac{1}{2}) = [\mathbf{16}_B \oplus \mathbf{16}_F]_{\downarrow}$ with $h^a = h^c = h^e = -\frac{7}{2}$ as follows

floor	SU(5/1)	field	helicity	
gnd >	$(0 \ 0 \ 0 \ 0 \ -\frac{1}{2})$	λ	$+\frac{1}{2}$	
1st >	$(0 \ 0 \ 0 \ 1 \ -\frac{1}{2})$	$\bar{5}\phi$	0	
2nd >	$(0 \ 0 \ 1 \ 0 \ +\frac{1}{2})$	$\bar{10}\lambda$	$-\frac{1}{2}$	(31)
3rd >	$(0 \ 1 \ 0 \ 0 \ +\frac{3}{2})$	$10A_{\mu}$	-1	
4th >	$(1 \ 0 \ 0 \ 0 \ +\frac{5}{2})$	$5\Psi_{\mu}$	$-\frac{3}{2}$	
5th >	$(0 \ 0 \ 0 \ 0 \ +\frac{7}{2})$	e_{μ}^a	-2.	

Therefore, the well-known supergravity multiplets $[\mathbf{32}_B \oplus \mathbf{32}_F]$ of the $N=5$ theory, which is given by

$$[\mathbf{1}_2 \oplus \bar{\mathbf{5}}_{3/2} \oplus \bar{\mathbf{10}}_1 \oplus \mathbf{10}_{1/2} \oplus \mathbf{5}_0 \oplus \mathbf{1}_{-1/2}] \oplus [\mathbf{1}_{1/2} \oplus \bar{\mathbf{5}}_0 \oplus \bar{\mathbf{10}}_{-1/2} \oplus \mathbf{10}_{-1} \oplus \mathbf{5}_{-3/2} \oplus \mathbf{1}_{-2}], \quad (32)$$

are exactly identified with two irreps of SU(5/1) in Eqs.(29) and (31). The other two typical reps $(0 \ 0 \ 0 \ 0 \ -\frac{5}{2})$ and $(0 \ 0 \ 0 \ 0 \ -\frac{3}{2})$ in Eqs.(27) and (28) are extra gravitino multiplets having the maximal helicity $\pm\frac{3}{2}$.

3.4 $N = 4$ Supergravity

The next stage of truncation is obtained from the branching rule $\text{SU}(5/1) \rightarrow \text{SU}(4/1) \otimes \text{U}^g(1)$ by using $\mathbf{P}_2(5)$ as follows

$$\begin{aligned} (0 \ 0 \ 0 \ 0 \ -\frac{7}{2}) &\rightarrow (0 \ 0 \ 0 \ -\frac{7}{2})(\frac{7}{2}) \oplus (0 \ 0 \ 0 \ -\frac{5}{2})(-\frac{1}{2}), \\ (0 \ 0 \ 0 \ 0 \ -\frac{1}{2}) &\rightarrow (0 \ 0 \ 0 \ -\frac{1}{2})(\frac{1}{2}) \oplus (0 \ 0 \ 0 \ +\frac{1}{2})(-\frac{7}{2}). \end{aligned} \quad (33)$$

Then, the typical rep $(0\ 0\ 0\ -\frac{7}{2}) = [\mathbf{16}_B \oplus \mathbf{16}_F]_{\uparrow}$ with $h^a = h^c = h^e = h^g = \frac{7}{2}$ contains the half of the supermultiplets of the $N=4$ theory as follows:

floor	SU(4/1)	field	helicity	
$ \text{gnd} >$	$(0\ 0\ 0\ -\frac{7}{2})$	e_{μ}^a	$+2$	
$ \text{1st} >$	$(0\ 0\ 1\ -\frac{7}{2})$	$\bar{4}\Psi_{\mu}$	$+\frac{3}{2}$	
$ \text{2nd} >$	$(0\ 1\ 0\ -\frac{5}{2})$	$6A_{\mu}$	$+1$	
$ \text{3rd} >$	$(1\ 0\ 0\ -\frac{3}{2})$	4λ	$+\frac{1}{2}$	
$ \text{4th} >$	$(0\ 0\ 0\ -\frac{1}{2})$	ϕ	$0,$	(34)

where the helicity quantum number is given by

$$h = -\frac{1}{84}h^a - \frac{1}{60}h^c - \frac{1}{40}h^e - \frac{1}{24}h^g - \frac{1}{6}h^h. \quad (35)$$

The complex conjugate rep $(0\ 0\ 0\ +\frac{1}{2}) = [\mathbf{16}_B \oplus \mathbf{16}_F]_{\downarrow}$ with $h^a = h^c = h^e = h^g = -\frac{7}{2}$ of Eq.(34) contains the remaining half of the supermultiplets as follows:

floor	SU(4/1)	field	helicity	
$ \text{gnd} >$	$(0\ 0\ 0\ +\frac{1}{2})$	ϕ	0	
$ \text{1st} >$	$(0\ 0\ 1\ +\frac{1}{2})$	$\bar{4}\lambda$	$-\frac{1}{2}$	
$ \text{2nd} >$	$(0\ 1\ 0\ +\frac{3}{2})$	$6A_{\mu}$	-1	
$ \text{3rd} >$	$(1\ 0\ 0\ +\frac{5}{2})$	$4\Psi_{\mu}$	$-\frac{3}{2}$	
$ \text{4th} >$	$(0\ 0\ 0\ +\frac{7}{2})$	e_{μ}^a	$-2.$	(36)

The other two typical reps $(0\ 0\ 0\ -\frac{5}{2})$ with $h^a = h^c = h^e = \frac{7}{2}$, and $h^g = -\frac{1}{2}$, and $(0\ 0\ 0\ -\frac{1}{2})$ with $h^a = h^c = h^e = -\frac{7}{2}$, and $h^g = +\frac{1}{2}$ stand for extra gravitino multiplets.

It seems appropriate to comment on two extra gravitino multiplets in $N=5$ theory. By using the matrix $\mathbf{P}_2(5)$, from the gravitino reps of $N=5$ theory we can easily generate the supermultiplets $[\mathbf{8}_B \oplus \mathbf{8}_F]$ of $D=4$, $N=4$ Yang-Mills theory, which are

$$[\mathbf{1}_1 \oplus \bar{\mathbf{4}}_{1/2} \oplus \mathbf{6}_0 \oplus \mathbf{4}_{-1/2} \oplus \mathbf{1}_{-1}], \quad (37)$$

in terms of $\text{SU}(4/1) \otimes \text{U}^g(1)$ reps as follows:

$$\begin{aligned} (0 \ 0 \ 0 \ 0 \ -\frac{5}{2}) &\rightarrow (0 \ 0 \ 0 \ -\frac{5}{2})(\frac{5}{2}) \oplus (0 \ 0 \ 0 \ -\frac{3}{2})(-\frac{3}{2}), \\ (0 \ 0 \ 0 \ 0 \ -\frac{3}{2}) &\rightarrow (0 \ 0 \ 0 \ -\frac{3}{2})(\frac{3}{2}) \oplus (0 \ 0 \ 0 \ -\frac{1}{2})(-\frac{5}{2}). \end{aligned} \quad (38)$$

Each *real* rep $(0 \ 0 \ 0 \ -\frac{3}{2})$ in Eq.(38) is exactly an Yang-Mills supermultiplet which has the contents as follows

floor	SU(4/1)	field	helicity	
gnd >	$(0 \ 0 \ 0 \ -\frac{3}{2})$	A_μ	+1	
1st >	$(0 \ 0 \ 1 \ -\frac{3}{2})$	$\bar{4}\lambda$	$+\frac{1}{2}$	
2nd >	$(0 \ 1 \ 0 \ -\frac{1}{2})$	6ϕ	0	
3rd >	$(1 \ 0 \ 0 \ +\frac{1}{2})$	4λ	$-\frac{1}{2}$	
4th >	$(0 \ 0 \ 0 \ +\frac{3}{2})$	A_μ	-1.	

(39)

As a result, each extra gravitino multiplet of $N=5$ theory splits into an gravitino and an Yang-Mills multiplets of $N=4$ theory in $\text{SU}(4/1) \otimes \text{U}^g(1)$ sub-superalgebra basis.

3.5 $N=3$ Supergravity

Next, let us carry out superalgebraic truncation from the $N=4$ to the $N=3$ theory. By using $\mathbf{P}_2(4)$, we get $N=3$ multiplets from the branching rule $\text{SU}(4/1) \rightarrow \text{SU}(3/1) \otimes \text{U}^i(1)$ as follows:

$$\begin{aligned} (0 \ 0 \ 0 \ -\frac{7}{2}) &\rightarrow (0 \ 0 \ -\frac{7}{2})(+\frac{7}{2}) \oplus (0 \ 0 \ -\frac{5}{2})(+\frac{1}{2}), \\ (0 \ 0 \ 0 \ +\frac{1}{2}) &\rightarrow (0 \ 0 \ +\frac{1}{2})(-\frac{1}{2}) \oplus (0 \ 0 \ +\frac{3}{2})(-\frac{7}{2}). \end{aligned} \quad (40)$$

The typical rep $(0 \ 0 \ -\frac{7}{2})$ with $h^a = h^c = h^e = h^g = h^i = \frac{7}{2}$ is $[\mathbf{4}_B \oplus \mathbf{4}_F]_\uparrow$, which contains the half of the supermultiplets of the $N=3$ theory as follows:

floor	SU(3/1)	field	helicity	
$ \text{gnd} >$	$(0 \ 0 \ -\frac{7}{2})$	e_μ^a	$+2$	
$ \text{1st} >$	$(0 \ 1 \ -\frac{7}{2})$	$\bar{3}\Psi_\mu$	$+\frac{3}{2}$	(41)
$ \text{2nd} >$	$(1 \ 0 \ -\frac{5}{2})$	$3A_\mu$	$+1$	
$ \text{3rd} >$	$(0 \ 0 \ -\frac{3}{2})$	λ	$+\frac{1}{2}$,	

where the helicity quantum number is given by

$$h = -\frac{1}{84}h^a - \frac{1}{60}h^c - \frac{1}{40}h^e - \frac{1}{24}h^g - \frac{1}{12}h^i - \frac{1}{4}h^j. \quad (42)$$

The complex conjugate rep $(0 \ 0 \ +\frac{3}{2})$ with $h^a = h^c = h^e = h^g = h^i = h^j = -\frac{7}{2}$ is $[\mathbf{4}_B \oplus \mathbf{4}_F]_\downarrow$, which contains the remaining half of the supermultiplets as follows:

floor	SU(3/1)	field	helicity	
$ \text{gnd} >$	$(0 \ 0 \ +\frac{3}{2})$	λ	$-\frac{1}{2}$	
$ \text{1st} >$	$(0 \ 1 \ +\frac{3}{2})$	$\bar{4}A_\mu$	-1	(43)
$ \text{2nd} >$	$(1 \ 0 \ +\frac{5}{2})$	$4\Psi_\mu$	$-\frac{3}{2}$	
$ \text{3rd} >$	$(0 \ 0 \ +\frac{7}{2})$	e_μ^a	-2 .	

As a result, the desired supergravity multiplets $[\mathbf{8}_B \oplus \mathbf{8}_F]$ of the $D=4$, $N=3$ theory, which is given by

$$[\mathbf{1}_2 \oplus \bar{\mathbf{3}}_{3/2} \oplus \mathbf{3}_1 \oplus \mathbf{1}_{1/2}] \oplus [\mathbf{1}_{-1/2} \oplus \bar{\mathbf{3}}_{-1} \oplus \mathbf{3}_{-3/2} \oplus \mathbf{1}_{-2}], \quad (44)$$

are identified with the two irreps in Eqs.(42) and (44).

On the other hand, the Yang-Mills multiplet of $N=4$ goes into Yang-Mills multiplets $[\mathbf{8}_B \oplus \mathbf{8}_F]$ of $N=3$ theory, which are

$$[\mathbf{1}_1 \oplus \bar{\mathbf{3}}_{1/2} \oplus \mathbf{3}_0 \oplus \mathbf{1}_{-1/2}] \oplus [\mathbf{1}_{1/2} \oplus \bar{\mathbf{3}}_0 \oplus \mathbf{3}_{-1/2} \oplus \mathbf{1}_{-1}], \quad (45)$$

in terms of two combined $\text{SU}(3/1) \otimes \text{U}^i(1)$ complex reps with $h^a = h^c = \pm\frac{7}{2}$, $h^e = h^g = \mp\frac{3}{2}$ such as

floor	SU(3/1)	field	helicity	
$ \text{gnd} >$	$(0 \ 0 \ -\frac{3}{2})$	A_μ	$+1$	
$ \text{1st} >$	$(0 \ 1 \ -\frac{3}{2})$	$\bar{3}\lambda$	$+\frac{1}{2}$	(46)
$ \text{2nd} >$	$(1 \ 0 \ -\frac{1}{2})$	3ϕ	0	
$ \text{3rd} >$	$(0 \ 0 \ +\frac{1}{2})$	χ	$-\frac{1}{2}$,	

and

floor	SU(3/1)	field	helicity	
$ \text{gnd} >$	$(0 \ 0 \ -\frac{1}{2})$	χ	$+\frac{1}{2}$	
$ \text{1st} >$	$(0 \ 1 \ -\frac{1}{2})$	$\bar{3}\phi$	0	(47)
$ \text{2nd} >$	$(1 \ 0 \ +\frac{1}{2})$	3λ	$-\frac{1}{2}$	
$ \text{3rd} >$	$(0 \ 0 \ +\frac{3}{2})$	A_μ	-1 .	

3.6 $N=2$ Supergravity

Next, let us carry out superalgebraic truncation from the $N=3$ to the $N=2$ theory. By using $\mathbf{P}_2(3)$, we get $N=2$ multiplets from the branching rule $\text{SU}(3/1) \rightarrow \text{SU}(2/1) \otimes \text{U}^k(1)$ as follows:

$$\begin{aligned}
(0 \ 0 \ -\frac{7}{2}) &\rightarrow (0 \ -\frac{7}{2})(+\frac{7}{2}) \oplus (0 \ -\frac{5}{2})(+\frac{3}{2}), \\
(0 \ 0 \ +\frac{3}{2}) &\rightarrow (0 \ +\frac{5}{2})(-\frac{7}{2}) \oplus (0 \ +\frac{3}{2})(-\frac{3}{2}).
\end{aligned}
\tag{48}$$

The typical rep $(0 \ -\frac{7}{2})$ with $h^a = h^c = h^e = h^g = h^i = h^k = \frac{7}{2}$ is $[\mathbf{2}_B \oplus \mathbf{2}_F]_\uparrow$, which contains the half of the supermultiplets of the $N=2$ theory as follows:

floor	SU(2/1)	field	helicity
$ \text{gnd} >$	$(0 \ -\frac{7}{2})$	e_μ^a	$+2$
$ \text{1st} >$	$(1 \ -\frac{7}{2})$	$2\Psi_\mu$	$+\frac{3}{2}$
$ \text{2nd} >$	$(0 \ -\frac{5}{2})$	A_μ	$+1$

(49)

where the helicity quantum number is given by

$$h = -\frac{1}{84}h^a - \frac{1}{60}h^c - \frac{1}{40}h^e - \frac{1}{24}h^g - \frac{1}{12}h^i - \frac{1}{4}h^k - \frac{1}{2}h^l. \quad (50)$$

The complex conjugate rep $(0 \ +\frac{3}{2})$ with $h^a = h^c = h^e = h^g = h^i = h^k = -\frac{7}{2}$ is $[\mathbf{2}_B \oplus \mathbf{2}_F]_\downarrow$, which contains the remaining half of the supermultiplets as follows:

floor	SU(2/1)	field	helicity
$ \text{gnd} >$	$(0 \ +\frac{5}{2})$	A_μ	-1
$ \text{2nd} >$	$(1 \ +\frac{5}{2})$	$2\Psi_\mu$	$-\frac{3}{2}$
$ \text{3rd} >$	$(0 \ +\frac{7}{2})$	e_μ^a	-2

(51)

As a result, the supergravity multiplets $[\mathbf{4}_B \oplus \mathbf{4}_F]$ of the $D=4$, $N=2$ theory, which is given by

$$[\mathbf{1}_2 \oplus \mathbf{2}_{3/2} \oplus \mathbf{1}_1] \oplus [\mathbf{1}_{-1} \oplus \mathbf{2}_{-3/2} \oplus \mathbf{1}_{-2}], \quad (52)$$

are identified with the two irreps in Eqs.(49) and (51).

On the other hand, the Yang-Mills multiplet of $N=3$ splits into one Yang-Mills multiplet and two matter multiplets. The Yang-Mills multiplet $[\mathbf{4}_B \oplus \mathbf{4}_F]$ of $N=2$ theory, which consists of

$$[\mathbf{1}_1 \oplus \mathbf{2}_{1/2} \oplus \mathbf{1}_0] \oplus [\mathbf{1}_0 \oplus \mathbf{2}_{-1/2} \oplus \mathbf{1}_{-1}], \quad (53)$$

in terms of two combined $\text{SU}(2/1) \otimes \text{U}^k(1)$ complex reps with $h^a = h^c = \pm\frac{7}{2}$, $h^e = h^g = \mp\frac{3}{2}$, $h^i = h^k = \pm\frac{3}{2}$, is given by

floor	SU(2/1)	field	helicity
$ \text{gnd} >$	$(0 \ -\frac{3}{2})$	A_μ	$+1$
$ \text{1st} >$	$(1 \ -\frac{3}{2})$	2λ	$+\frac{1}{2}$
$ \text{2nd} >$	$(0 \ -\frac{1}{2})$	ϕ	0

(54)

and

floor	SU(2/1)	field	helicity
$ \text{gnd} >$	$(0 \ +\frac{1}{2})$	ϕ	0
$ \text{1st} >$	$(0 \ +\frac{1}{2})$	2λ	$-\frac{1}{2}$
$ \text{2nd} >$	$(0 \ +\frac{3}{2})$	A_μ	$-1.$

(55)

On the other hand, each matter multiplet $[\mathbf{2}_B \oplus \mathbf{2}_F]$ of $N=2$ theory, which is

$$[\mathbf{1}_{1/2} \oplus \mathbf{2}_0 \oplus \mathbf{1}_{-1/2}], \quad (56)$$

in terms of $\text{SU}(2/1) \otimes \text{U}^k(1)$ real rep with $h^a = h^c = \pm \frac{7}{2}, h^e = h^g = \mp \frac{3}{2}, h^i = h^k = \pm \frac{3}{2}$, is given by

floor	SU(2/1)	field	helicity
$ \text{gnd} >$	$(0 \ -\frac{1}{2})$	λ_+	$+\frac{1}{2}$
$ \text{1st} >$	$(1 \ -\frac{1}{2})$	2Φ	0
$ \text{2nd} >$	$(0 \ +\frac{1}{2})$	λ_-	$-\frac{1}{2}.$

(57)

3.7 $N=1$ Supergravity

Finally, let us carry out superalgebraic truncation from the $N=2$ to the $N=1$ theory. In this case, although all gauge symmetries are completely broken except $\text{U}(1)$ helicity, let us consider the $N=1$ multiplets comparing with $N=2$ rep for the sake of uniform notation. By using $\mathbf{P}_2(2)$, we get $N=1$ multiplets from $N=2$ multiplets by using the

branching rule $SU(2/1)(\supset SU(2) \otimes U^l(1)) \rightarrow U^m(1) \otimes U^l(1)$. From the full weight system of the typical rep $(0 - \frac{7}{2})$ with $h^a = h^c = h^e = h^g = h^i = h^k = \frac{7}{2}$, one can obtain the halves of the graviton and extra gravitino supermultiplets of the $N=1$ theory, which are

$$[\mathbf{1}_2 \oplus \mathbf{1}_{3/2}]_{\uparrow} \oplus [\mathbf{1}_{3/2} \oplus \mathbf{1}_1], \quad (58)$$

as follows:

floor	SU(2/1)	$U^m(1) \otimes U^l(1)$	field	helicity	
$ \text{gnd} \rangle$	$(0 - \frac{7}{2})$	$(+\frac{7}{2})(-7)$	e_{μ}^a	$+2$	
$ \text{1st} \rangle$	$(1 - \frac{7}{2})$	$(+\frac{7}{2})(-6)$	Ψ_{μ}	$+\frac{3}{2}$	(59)
	$(-1 - \frac{5}{2})$	$(+\frac{5}{2})(-6)$	χ_{μ}	$+\frac{3}{2}$	
$ \text{2nd} \rangle$	$(0 - \frac{5}{2})$	$(+\frac{5}{2})(-5)$	A_{μ}	$+1$,	

where the helicity quantum number h is the same form as that of Eq.(50). Although h^m quantum number do not contribute to the helicity, it is important to classify the supermultiplets. From the complex conjugate rep $(0 + \frac{3}{2})$ with $h^a = h^c = h^e = h^g = h^i = h^k = -\frac{7}{2}$, we obtain

$$[\mathbf{1}_{-1} \oplus \mathbf{1}_{-3/2}]_{\downarrow} \oplus [\mathbf{1}_{-3/2} \oplus \mathbf{1}_{-2}], \quad (60)$$

which are the remaining halves of the supermultiplets as follows:

floor	SU(2/1)	$U^m(1) \otimes U^l(1)$	field	helicity	
$ \text{gnd} \rangle$	$(0 + \frac{5}{2})$	$(-\frac{5}{2})(5)$	A_{μ}	-1	
$ \text{1st} \rangle$	$(1 + \frac{5}{2})$	$(-\frac{5}{2})(6)$	χ_{μ}	$-\frac{3}{2}$	(61)
	$(-1 + \frac{7}{2})$	$(-\frac{7}{2})(6)$	Ψ_{μ}	$-\frac{3}{2}$	
$ \text{2nd} \rangle$	$(0 + \frac{7}{2})$	$(-\frac{7}{2})(7)$	e_{μ}^a	-2 .	

As a result, the desired graviton multiplet $[\mathbf{2}_B \oplus \mathbf{2}_F]$ of the $D=4$, $N=1$ theory, which is given by

$$[\mathbf{1}_2 \oplus \mathbf{1}_{3/2}] \oplus [\mathbf{1}_{-3/2} \oplus \mathbf{1}_{-2}], \quad (62)$$

is obtained from two irreps in Eqs.(59) and (61) by removing the extra gravitino part.

On the other hand, the Yang-Mills multiplet of $N=2$ splits into Yang-Mills and matter multiplets of $N=1$ theory. As a result, we have the Yang-Mills multiplet $[\mathbf{2}_B \oplus \mathbf{2}_F]$ of $N=1$ theory, which is

$$[\mathbf{1}_1 \oplus \mathbf{1}_{1/2}] \oplus [\mathbf{1}_{-1/2} \oplus \mathbf{1}_{-1}], \quad (63)$$

and the matter multiplet $[\mathbf{2}_B \oplus \mathbf{2}_F]$, which is

$$[\mathbf{1}_{1/2} \oplus \mathbf{1}_0] \oplus [\mathbf{1}_0 \oplus \mathbf{1}_{-1/2}], \quad (64)$$

in terms of the full weight system of two combined $SU(2/1)$ complex reps of $N=2$ theory with $h^a = h^c = \pm \frac{7}{2}, h^e = h^g = \mp \frac{3}{2}, h^i = h^k = \pm \frac{3}{2}$ as follows

floor	SU(2/1)	$U^m(1) \otimes U^l(1)$	field	helicity	
$ \text{gnd} >$	$(0 \ -\frac{3}{2})$	$(\frac{3}{2})(-3)$	A_μ	$+1$	
$ \text{1st} >$	$(1 \ -\frac{3}{2})$	$(\frac{3}{2})(-2)$	λ	$+\frac{1}{2}$	(65)
	$(1 \ +\frac{1}{2})$	$(\frac{1}{2})(-2)$	χ	$+\frac{1}{2}$	
$ \text{2nd} >$	$(0 \ -\frac{1}{2})$	$(\frac{1}{2})(-1)$	ϕ	$0,$	

and

floor	SU(2/1)	$U^m(1) \otimes U^l(1)$	field	helicity	
$ \text{gnd} >$	$(0 \ +\frac{1}{2})$	$(-\frac{1}{2})(1)$	ϕ	0	
$ \text{1st} >$	$(1 \ +\frac{1}{2})$	$(-\frac{1}{2})(2)$	χ	$-\frac{1}{2}$	(66)
	$(-1 \ +\frac{3}{2})$	$(-\frac{3}{2})(2)$	λ	$-\frac{1}{2}$	
$ \text{2nd} >$	$(0 \ +\frac{3}{2})$	$(-\frac{3}{2})(3)$	A_μ	$-1.$	

On the other hand, we can also obtain the matter multiplet $[\mathbf{2}_B \oplus \mathbf{2}_F]$ of $N=1$ theory from the matter multiplet of $N=2$ theory, which is

$$[\mathbf{1}_{1/2} \oplus \mathbf{2}_0 \oplus \mathbf{1}_{-1/2}], \quad (67)$$

in terms of the full weight system of $SU(2/1)$ real rep with $h^a = h^c = \pm \frac{7}{2}, h^e = h^g = \mp \frac{3}{2}, h^i = h^k = \pm \frac{3}{2}$ as follows

floor	$SU(2/1)$	$U^m(1) \otimes U^l(1)$	field	helicity	
$ \text{gnd} >$	$(0 \ -\frac{1}{2})$	$(+\frac{1}{2})(-1)$	λ_+	$+\frac{1}{2}$	
$ \text{1st} >$	$(1 \ -\frac{1}{2})$	$(+\frac{1}{2})(0)$	Φ	0	(68)
	$(-1 \ +\frac{1}{2})$	$(-\frac{1}{2})(0)$	Φ	0	
$ \text{2nd} >$	$(0 \ +\frac{1}{2})$	$(-\frac{1}{2})(1)$	λ_-	$-\frac{1}{2}$	

IV. Conclusion

In conclusion, we have studied $D=4$, $N=8$ supergravity in the context of $SU(8/1)$ Lie superalgebra. We have constructed suitable general projection matrix, and used this matrix to obtain all possible regular maximal branching patterns in terms of Kac-Dynkin weight techniques. Then, we have shown that consistent successive superalgebraic truncations from the $D=4$, $N=8$ maximal theory to the $D=4$, $N=7, 6, \dots, 1$ theories can be systematically realized as sub-algebra chains of the $SU(8/1)$ superalgebra. As results, we have identified all possible supermultiplets of $N=8, 7, \dots, 1$ theories, which have been classified in terms of super-Poincaré algebra by Strathdee, with irreps of $SU(N/1)$ superalgebra. Finally, we hope that our branching technique will provide a deeper understanding of the structure of the supersymmetric systems and help finding new supersymmetric theories.

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